Power factor methodology report

Methodology for the power factor resolution algorithm

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Abstract

This document describes the methodology used to compute the power factor at the inverter output used by the pvDesign software. The goal is to calculate the required cosine of phi value at the output of the inverters to meet some given grid requirements, such as a specific power factor value at the delivery point.

Contents

1 Introduction

In this document, a methodology to calculate the power factor after applying a chain of losses is described. This methodology will be used to program an algorithm which will be able to compute the power factor at the start of the chain to meet a required value at the end.

The power transmission system model, which details the components in the aforementioned chain of losses, is described in Section 3.

The chain of losses will be composed of cables and transformers. The mathematical models describing the losses incurred in the components of the chain are described in Section 2.

Some simplifications will have to be made in order to use the algorithm in the context of the pvDesign software. The simplifications can be separated in two categories:

- Plant sizing simplifications. In pvDesign, when computing the required power factor at the inverter output, the DC capacity of the plant is unknown, so it must be estimated. Furthermore, because at this stage there is no information regarding the power station sizing or the cables which will be used (both may be dynamic depending on the plant design), it is not possible to compute the exact losses of the transmission systems up to the substation. For this reason, all the cables (excluding the high voltage line) and the power station transformers are assumed to be equal.

- Equipment model simplifications. The cables are assumed to only yield active power losses. The transformers are assumed to operate at a fixed power factor, which is close to full load. The power factor is referred to the rated power of the transformers.

Because of this simplifications, the result of this model can only be considered an estimate. In pvDesign, when the energy yield is calculated, the cosine of phi at the inverter output resulting from this model will be used as a fixed output value at the inverter resolution. Because the energy yield module has the full sizing data about the plant, and because it compute the power

factor dynamically depending on the production, the resulting power factor at the end of the transmission system calculated by the energy yield module may slightly differ from the desired value. Finally, the algorithm itself will be an iterative equation resolution algorithm (such as the Newton-Raphson method). While it would be possible to simplify the equations resulting from the models and get an explicit solution, we decided against this option in order to maintain better modularity in the implementation in pvDesign.

2   Mathematical models

In this section, the models used to compute the different inputs and losses are detailed.

2.1   Photovoltaic plant sizing

The plant sizing calculation consists in computing the rated power of every component in the chain of losses, and the number of times each component is repeated. All of this results depend directly on the plant DC power capacity. In the pvDesign software, there are two distinct ways in which the plant capacity can be defined.

- In maximum capacity mode, the software will use as much of the available surface area as possible to maximize the photovoltaic plant rated power.
- In specific capacity mode, the number of inverters will be predefined, and so the photovoltaic plant rated power will be fixed.

The first step in either mode is to calculate the rated power of the photovoltaic plant. If the specific capacity mode is selected, then the rated power of the plant is already implicit, and will be calculated using the number of inverters and the rated power of each. However, in maximum capacity mode it must be approximated using (1), as the real capacity value can only be known after performing a full simulation.

$$S_{\text{plant}} = \frac{W_{\text{dc}}}{r_{\text{dcac}} \cdot \cos \varphi_{\text{inv}}} \tag{1}$$

Where:

- S_{plant} is the rated power of the photovoltaic plant, in [VA].
- W_{dc} is the total DC capacity of the plant, in [W], calculated using equation (2) for maximum capacity mode.
- r_{dcac} is the DC/AC ratio, referred to the inverter active power.
- $\cos \varphi_{\text{inv}}$ is the power factor at the inverter output.

The DC capacity can be estimated using the net available surface area, and the surface area of each structure (including pitch distance), as shown in equation (2).

$$W_{\text{dc}} = \frac{a_{\text{site}}}{d_{\text{pitch}} \cdot l_{\text{struct}}} \cdot W_{\text{stc}} \cdot n_{\text{str}} \tag{2}$$

Where:

- W_{dc} is the total DC capacity of the plant, in [W].
- a_{site} is the net available surface area, in [m^2].
- d_{pitch} is the pitch distance between rows, in [m].

- l_{struct} is an estimation of the structure length along its axis, calculated using equation (3).
- W_{stc} is the photovoltaic module rated power in STC conditions, in [W].
- n_{str} is the number of modules per string.

$$l_{\text{struct}} = \frac{n_{\text{str}}}{n_{\text{mod, p}}} \cdot d_{\text{mod}} \tag{3}$$

Where:

- l_{struct} is an estimation of the structure length along its axis.
- n_{str} is the number of modules per string.
- $n_{\text{mod, p}}$ is the number of modules stacked in the direction perpendicular to the structure axis. For instance, 1 in "1V" trackers, or 3 in "3H" fixed structures.
- d_{mod} is the relevant photovoltaic module dimension, which depends on whether the module orientation is portrait or landscape.

Once the photovoltaic plant DC capacity has been calculated, the number of inverters and power stations can be calculated using equations (4) and (5), respectively.

$$n_{\text{inv}} = \frac{S_{\text{plant}}}{S_{\text{inv}}} \tag{4}$$

Where:

- n_{inv} is the total number of inverters in the photovoltaic plant.
- S_{plant} is the rated power of the photovoltaic plant, calculated using equation (1).
- S_{inv} is the rated power of the inverter, in [VA].

$$n_{\text{ps}} = \frac{S_{\text{plant}}}{S_{\text{inv}} \cdot n_{\text{inv per ps}}} \tag{5}$$

- n_{ps} is the total number of power stations in the photovoltaic plant.
- S_{plant} is the rated power of the photovoltaic plant, calculated using equation (1).
- S_{inv} is the rated power of the inverter, in [VA].
- $n_{\text{inv per ps}}$ is the number of inverters per power station.

2.2   Transformer losses

A transformer raises the voltage of the AC electrical power at its input. While the efficiency of modern transformers is very high, in operation there are losses small active power losses, and more importantly for the purposes of this model, there is a non-insignificant reactive power consumption (because the transformer is an inductive component). These losses can be classified in two categories [1]:

- Core losses (iron loss), due to hysteresis and eddy-current in the core, a constant loss at all loads.
- Winding loss (copper loss), due to Joule heating in the conductor, dependent on the square of the load current.

To compute the winding loss, the load factor is used to represent the load current. The load factor is the division of the power at the transformer input by the rated power of the transformer. It is

estimated to take a fixed value of 0.88 for the calculation of losses. This value was found to be a good compromise between overestimating the losses (if the load factor was conservatively set to one), and matching the results of the energy simulation.

The active power losses of each type are calculated using equations (6) and (7).

$$W_{\text{iron}} = F_{\text{iron}} \cdot S_{\text{rated}} \tag{6}$$

Where:

- W_{iron} is the transformer iron loss.
- F_{iron} is the iron loss, in parts per one.
- S_{rated} is the transformer rated power, in [VA].

$$W_{\text{copper}} = F_{\text{copper}} \cdot S_{\text{rated}} \cdot x^2 \tag{7}$$

Where:

- W_{copper} is the transformer copper loss.
- F_{copper} is the copper loss, in parts per one.
- S_{rated} is the transformer rated power, in [VA].
- x is the load factor, which takes a constant value of 0.88.

The reactive power losses are calculated using the model described by Joksimovic [2]. The model uses an empirical approximation to calculate the no load current shown in equation (9), and the short circuit voltage to calculate the copper component reactive consumption.

The iron component of the reactive power consumption is calculated using equation (8).

$$Q_{\text{iron}} = \sqrt{\left(\frac{i_0}{100} \cdot S_{\text{rated}}\right)^2 - W_{\text{iron}}^2} \tag{8}$$

Where:

- Q_{iron} is the reactive consumption in the transformer due to the iron losses, in [VAr].
- i_0 is the no load current, in percentage value, calculated using (9).
- S_{rated} is the transformer rated power, in [VA].
- W_{iron} is the transformer iron loss, calculated using equation (6).

The no load current can be estimated using equation (9). In Joksimovic [2], this equation is referred to the transformer rated power in MVA.

$$i_0 = 0.0421 \cdot \left(\log(S_{\text{rated}} \cdot 10^{-6})\right)^2 - 0.4384 \cdot \log(S_{\text{rated}} \cdot 10^{-6}) + 1.6064 \tag{9}$$

Where:

- i_0 is the no load current, in percentage value.
- S_{rated} is the transformer rated power, in [VA].

The copper component of the reactive power consumption is calculated using equation (10).

$$Q_{\text{copper}} = x^2 \cdot \sqrt{\left(\frac{u_{\text{sc}}}{100} \cdot S_{\text{rated}}\right)^2 - W_{\text{copper}}^2} \tag{10}$$

Where:

- Q_{copper} is the reactive consumption in the transformer due to the copper losses, in [VAr].
- x is the load factor, which takes a constant value of 0.88.
- u_{sc} is the short circuit voltage percentage.
- S_{rated} is the transformer rated power, in [VA].
- W_{copper} is the transformer copper loss, calculated using equation (7).

The short circuit voltage percentage, u_{sc}, takes a value of 8 for power station transformers and 12.5 for the substation transformer [3].

The active power loss in the transformer can be calculated using equation (11), and the reactive power consumption using equation (12).

$$W_{\text{t}} = W_{\text{iron}} + W_{\text{copper}} \tag{11}$$

Where:

- W_{t} is the total transformer active power loss.
- W_{iron} is the transformer iron loss, calculated using equation (6).
- W_{copper} is the transformer copper loss, calculated using equation (7).

$$Q_{\text{t}} = Q_{\text{iron}} + Q_{\text{copper}} \tag{12}$$

Where:

- Q_{t} is the total transformer reactive power consumption.
- Q_{iron} is the transformer iron reactive consumption, calculated using equation (8).
- Q_{copper} is the transformer copper reactive consumption, calculated using equation (10).

2.3   Cable losses

The ohmic losses incurred in the low voltage and high voltage cables are calculated using equation (13). The reactive power consumption can be deemed insignificant for the purposes of this model.

$$W_{\text{c}} = S_{\text{rated}} \cdot u_{\text{loss}} \tag{13}$$

Where:

- W_{c} is the power loss incurred in a cable, in [VA].
- S_{rated} is the rated power of the cable, in [VA].
- u_{loss} is the maximum admissible voltage drop in the cable, in parts per one.

For the medium voltage cables, the reactive power can be taken into account in order to improve the accuracy of the model. The relationship between the resistance and reactance of the MV cables is estimated to determine the inductive effect in the losses.

The resistivity of the MV cable is calculated according to equation (14):

$$\rho = \rho_{20} \cdot (1 + \alpha(\theta_{\text{i}} - 20)) \tag{14}$$

Where:

- ρ is the conducting material resistivity at the cable's insulator maximum operational temperature, in $[\Omega m^2/m]$.

- ρ_{20} is the conducting material resistivity at 20ºC, in $[\Omega m^2/m]$, which is equal to $1/56 \cdot 10^{-6} \ \Omega m^2/m$ for copper and $1/35 \cdot 10^{-6} \ \Omega m^2/m$ for aluminium.

- α is a parameter that depends on the type of material used equal to $0.00392 \ ^\circ C^{-1}$ for copper and $0.00403 \ ^\circ C^{-1}$ for aluminium.

- θ_i is the maximum allowable conductor temperature, in [ºC] determined by the maximum operational insulator temperature in normal operation.

The resistance of the MV cable is then calculated using equation (15).

$$R_{\mathrm{mv}} = \frac{\rho(\theta_i)}{\Phi} \tag{15}$$

Where:

- R_{mv} is the resistance of the MV cable in $[\Omega/m]$.

- $\rho(\theta)$ is the resistivity of the conductor at temperature θ in $\left[\dfrac{\Omega}{m^2 \cdot m}\right]$

- Φ is the section of the conductor in $[m^2]$. A value of 400 mm^2 was selected, as it is a fairly common value for medium voltage cables.

The inductive reactance of the cables can be calculated based on its inductance, calculated from equation (16). As with the resistance, some geometrical considerations had to be made in order to obtain an estimated value for the reactance per unit length.

$$L = \left(K + 0.2 \cdot \ln \frac{2 \cdot S}{d}\right) \cdot 10^{-6} \tag{16}$$

Where:

- L is the inductance of the cable in [H/m].

- K is a constant that depends on the number of wires in the cable. A value of 0.05 is used, which corresponds to solid conductors [4].

- S is the axial spacing between cables in trefoil in [m]. A value of two times the diameter was selected, assuming an insulation thickness equal to the conductor's radius. The cables in trefoil are considered to not have an extra separation between each other.

- d is the diameter of the cable in [m].

The reactance is then calculated using equation (17):

$$X_L = 2\pi f L \tag{17}$$

Where:

- X_L is the reactance of the cable, in $[\Omega/m]$.

- f is the frequency, in [Hz].

- L is the inductance calculated from equation (16), in [H/m].

Knowing the estimated values of the resistance and the reactance per unit length of the MV cable and the admissible voltage drop in parts per unit, the active and reactive losses can be calculated
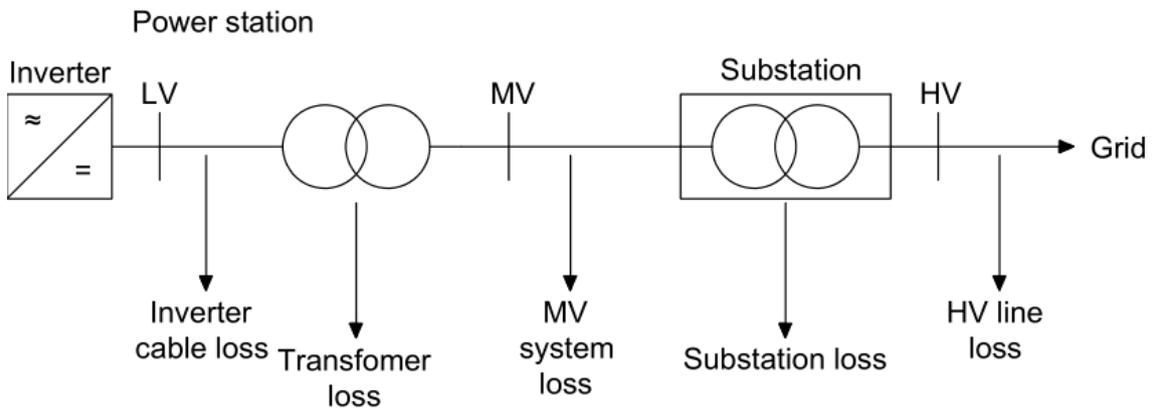
Figure 1: Diagram of the AC transmission system model

respectively with equations (18) and (19). Note that if the cosine of phi is equal to 1, the inductive losses will be equal to zero, meaning that equation (13) will be used instead.

$$W_c = \sqrt{\frac{(S_{\text{rated}} \cdot u_{\text{loss}})^2}{1 + \left(\dfrac{R \cos \varphi_{\text{inv}}}{X \sin \varphi_{\text{inv}}}\right)^2} \cdot \frac{R \cos \varphi_{\text{inv}}}{X \sin \varphi_{\text{inv}}}} \tag{18}$$

$$Q_c = \sqrt{\frac{(S_{\text{rated}} \cdot u_{\text{loss}})^2}{1 + \left(\dfrac{R \cos \varphi_{\text{inv}}}{X \sin \varphi_{\text{inv}}}\right)^2}} \tag{19}$$

Where:

- W_c is the active power loss incurred in the cable, in [VA].
- Q_c is the reactive power loss incurred in the cable, in [VA].
- S_{rated} is the rated power of the cable, in [VA].
- R is the resistance per unit length of the cable, in [Ω/m].
- X is the inductive reactance per unit length of the cable, in [Ω/m].
- φ_{inv} is the angle phi at the inverter output, in [rad].

3 Power transmission system model

The model of the power transmission system is shown in Figure 1. It is composed of the tri-phase cables connecting each component to the next, and the transformers which raise the voltage. Additionally, capacitor banks can be added at the input of the substation to compensate for the inductive losses of the other elements.

In order to compute the loss incurred in each component, the loss value is multiplied by the number of repetitions in the photovoltaic plant. The total active and reactive power losses are shown in equations (20) and (21), respectively.

$$W_{\text{total}} = W_{c,\text{inv}} \cdot n_{\text{inv}} + W_{c,\text{mv}} \cdot n_{\text{ps}} + W_{c,\text{hv}} + W_{t,\text{ps}} \cdot n_{\text{ps}} + W_{t,\text{sub}} \tag{20}$$

Where:

- W_{total} is the total active power loss in the system, in [W]
- $W_{\text{c, inv}}$ is the power loss incurred in a single inverter cable, calculated using equation (13), with the rated power of an inverter as the cable rated power.
- n_{inv} is the total number of inverters in the photovoltaic plant, calculated using equation (4)
- $W_{\text{c, mv}}$ is the power loss incurred in a single MV cable, calculated using equation (13), with the rated power of a power station as the cable rated power.
- n_{ps} is the total number of power stations in the photovoltaic plant, calculated using equation (5)
- $W_{\text{c, hv}}$ is the power loss incurred in the high voltage line to the grid, calculated using equation (13), with the plant rated power as the cable rated power.
- $W_{\text{t, ps}}$ is the active power loss incurred in the power station transformers, calculated using equation (11) at power station rated power.
- $W_{\text{t, sub}}$ is the active power loss incurred in the substation transformer, calculated using equation (11) at substation rated power.

$$Q_{\text{total}} = Q_{\text{t, ps}} \cdot n_{\text{ps}} + Q_{\text{t, sub}} + Q_{\text{c,mv}} \tag{21}$$

Where:

- Q_{total} is the total reactive power consumption in the system, in [VA].
- $Q_{\text{t, ps}}$ is the reactive power consumption incurred in the power station transformers, calculated using equation (12) at power station rated power.
- n_{ps} is the total number of power stations in the photovoltaic plant, calculated using equation (5).
- $Q_{\text{t, sub}}$ is the reactive power consumption incurred in the substation transformer, calculated using equation (12) at substation rated power.
- $Q_{\text{c,mv}}$ is the reactive power consumption incurred in the medium voltage cables, calculated using equation (19).

Finally, the angle phi at the delivery point can be calculated using equation (22). At the input of the chain of losses, the active and reactive power values can be calculated, using the plant rated power (which results from the number of inverters) and the phi angle at the inverter output.

$$\varphi_d = \arctan\left(\frac{S_{\text{plant}} \cdot \sin \varphi_{\text{inv}} - Q_{\text{total}}}{S_{\text{plant}} \cdot \cos \varphi_{\text{inv}} - W_{\text{total}}}\right) \tag{22}$$

Where:

- φ_d the angle phi at the delivery point.
- S_{plant} is the rated power of the photovoltaic plant, calculated using equation (1).
- φ_{inv} is the angle phi at the inverter output.
- Q_{total} is the total reactive power consumption in the system, calculated using equation (21).
- W_{total} is the total active power loss in the system, calculated using equation (20).

In Figure 2, a phasor representation of the chain of losses is shown (the losses are not to scale).
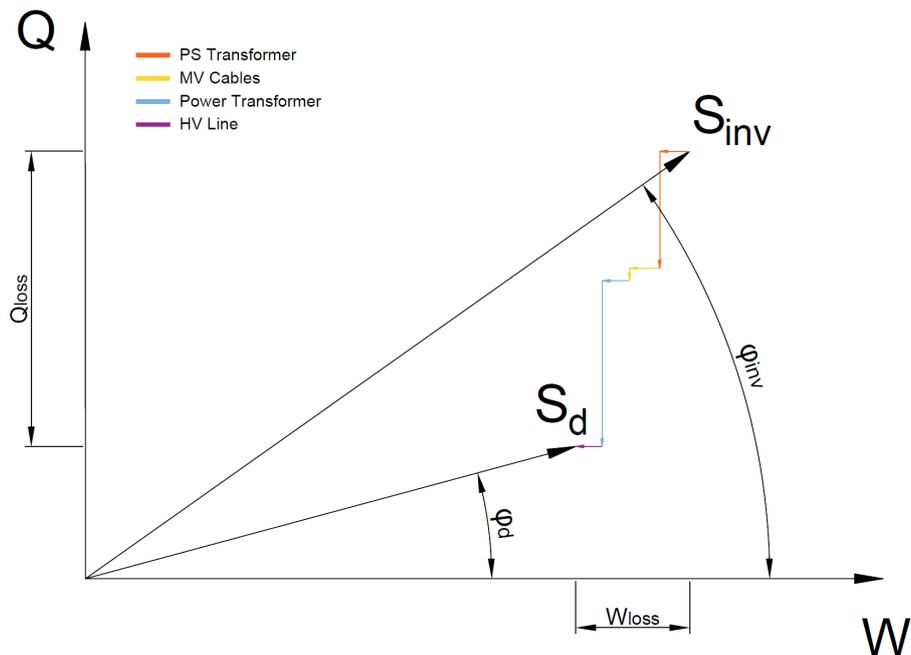
Figure 2: Phasor representation of the phi angle at the delivery point (not to scale)

4   Capacitor Banks

Capacitor banks are electrical devices aimed at improving the power factor of an electrical system, in this case the PV plant. Their objective is to compensate the inductive losses of the different elements present in the plant to improve the power factor at the inverters, resulting in a decrease of the required number of inverters.

The installation of the capacitor banks takes place at the input of the substation, specifically in the medium voltage switchgears (see Figure 3). This is also applicable to switching and breaking stations despite not having a power transformer.
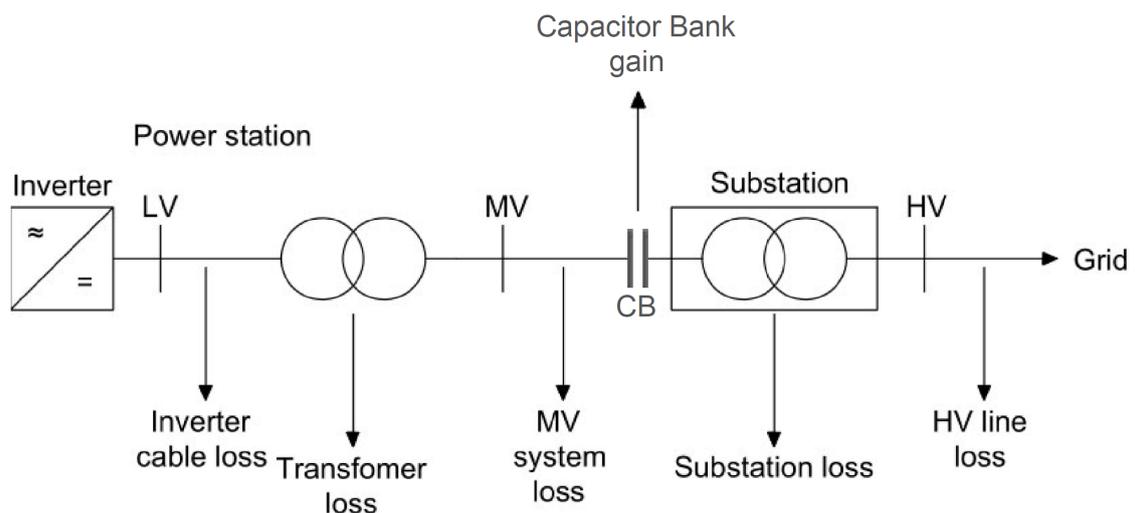

Figure 3: Diagram of the AC transmission system model with capacitor bank

The sizing of capacitor banks depends on the systems that need to be compensated by it. A designer could size a capacitor bank with a rated reactive power big enough to compensate the inductive losses induced by all the elements in the plant. The equation that describes the capacitive reactance of the sized capacitor bank is presented in Equation 23:

$$Q_{cb} = -\sum Q_{ind,loads} \tag{23}$$

Where:

- Q_{cb} is the total reactive power provided by the capacitor bank, in [VA]
- $Q_{ind,loads}$ is the total reactive power consumed by the inductive loads of the plant. The inductive loads come from the elements from Equation 21 that will be compensated by the capacitors.

An example of a phasor diagram that compensates all the systems in the PV plant is shown in Figure 4. The capacitor bank is sized to compensate the entire inductive losses of the cables and transformers, resulting in a power factor closer to 1 at the inverters. This happens due to the resistive losses that produce a horizontal shift in the phasor diagram between the delivery point and the inverters.
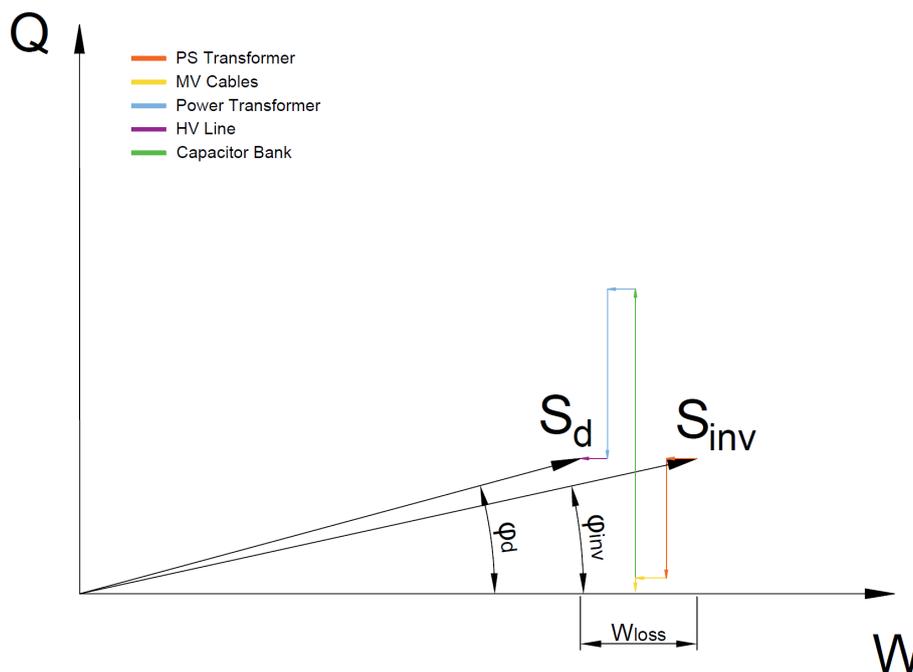

Figure 4: Phasor representation of the phi angle at the delivery point with capacitor banks compensating all the systems

Figure 5 presents the phasor diagram of a design where the capacitor bank is only sized to compensate the inductive losses generated by the power transformer. The remaining inductive losses result in a higher phase shift (φ) at the inverters, lowering the power factor required.
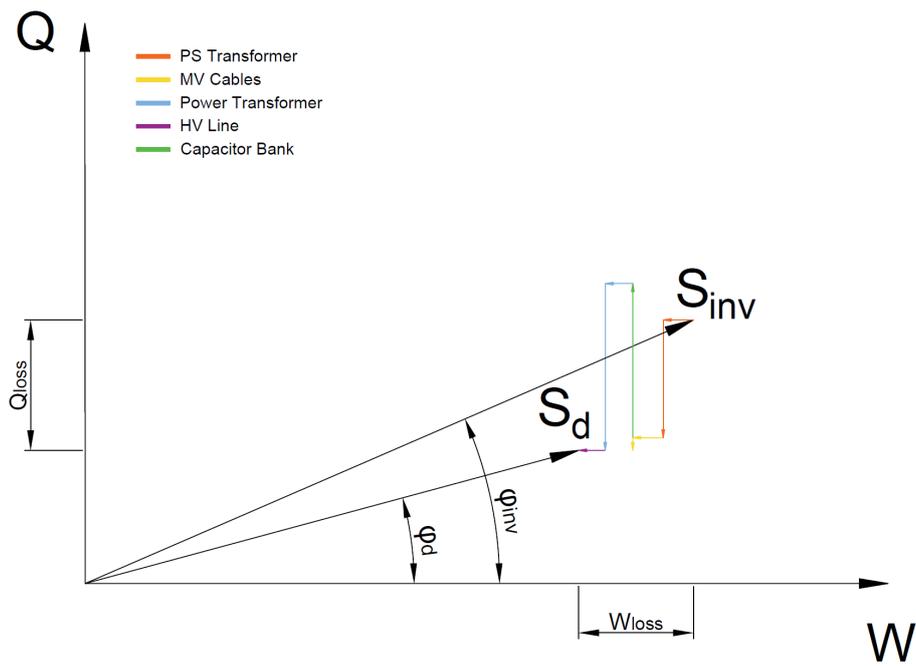
Figure 5: Phasor representation of the phi angle at the delivery point with capacitor banks compensating the power transformer

Figure 6 shows a case where the desired power factor at the delivery point is equal to 1, while maintaining the capacitor bank's sizing strategy of compensating all systems.
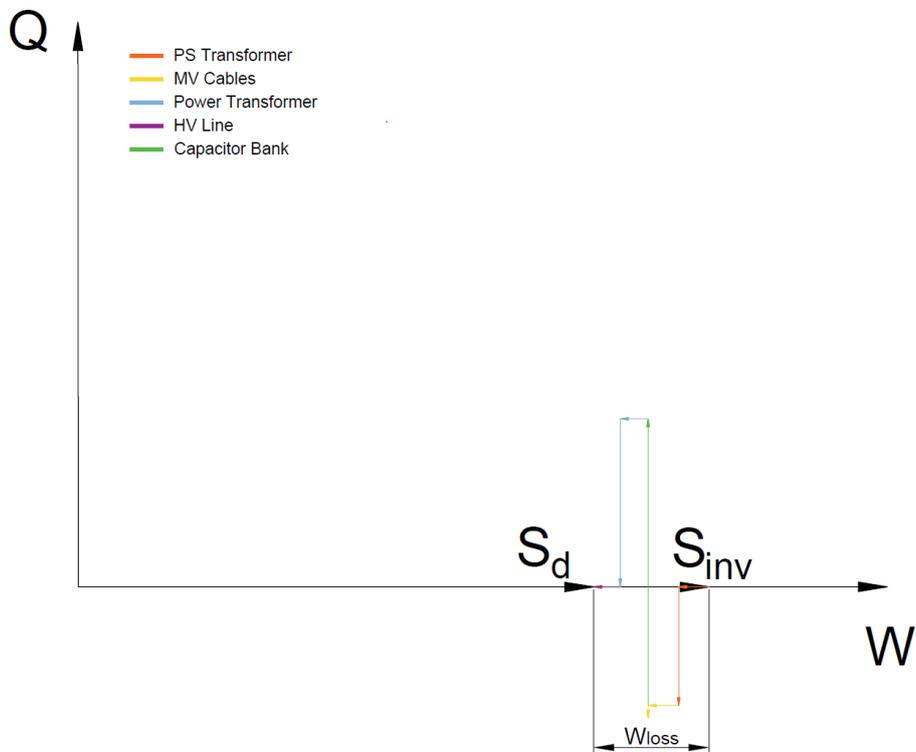

Figure 6: Phasor representation of the phi angle at the delivery point with capacitor banks compensating the all the systems with a required power factor of 1

While the strategy followed by pvDesign aims at compensating the inductive losses caused by the different systems in the PV plant, there are other strategies that could be used when sizing the capacitor banks. Manually selecting the MVAr rating of the capacitors or sizing the capacitors to reach a specific power factor at the inverters while maintaining the desired power factor at the delivery point are two alternative strategies that are not included in pvDesign.

5 Algorithm

As explained in Section 1, to solve this problem there are two possible approaches: either reduce the equations to the point where the cosine of phi at the inverter output can be deduced, or use an iterative resolution process.

While the first approach may be faster in terms of compute time, it yields a more complicated equation system, and is much harder to tweak in regards to the components included in the chains of losses. That is, to add or remove components, the whole equation must be re-written, which makes the implementation much harder to maintain.

Using an iterative resolution process yields much better code in terms of modularity, and is also easier to modify or tweak, as whole components can be swapped out for testing or for implementing different mathematical models.

The iterative process will consist in finding the roots (intersection of $f(x)$ with the horizontal axis, such that $f(x = \text{solution}) = 0$) of the equation (24).

$$f_{\text{to zero}} = \varphi_{\text{d, desired}} - \varphi_d(\varphi_{\text{inv}}) \tag{24}$$

Where:

- $f_{\text{to zero}}$ is the function for which the roots will be found.
- $\varphi_{\text{d, desired}}$ is the desired value of the cosine of phi at the output of the chain of losses.
- $\varphi_d(\varphi_{\text{inv}})$ is the value of the cosine of the phi at the output of the chain of losses, calculated using equation (22), for the cosine of phi of the inverter output (iteration value).

References

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[3] International Electrotechnical Commission, *IEC 60076-5:2006 Power transformers - Part 5: Ability to withstand short circuit.* 2006.

[4] BICC Cables Ltd, *Electric Cables Handbook, 3rd Edition.* 1997.

Appendix A   Numerical examples

In order to better understand how the model works, and the order of magnitude of the results, some simple numerical examples can be useful.

In this example, the DC capacity has already been calculated using equation (2). The case of study will have the following inputs:

- DC capacity $W_{dc} = 500\,\text{MVA}$.
- DC/AC ratio $r_{dcac} = 1.2$.
- Inverter rated power $S_{inv} = 1500\,\text{kVA}$.
- Number of inverters per power station $n_{\text{inv per ps}} = 2$.
- Iron loss factor $F_{iron} = 0.001$, for all transformers.
- Copper loss factor $F_{copper} = 0.005$, for all transformers.
- Inverter cable losses can be disregarded (central inverter system).
- Medium voltage system cables will be sized with a maximum admissible voltage drop of $u_{loss} = 0.005$.
- The high voltage line to the grid will be sized with a maximum admissible voltage drop of $u_{loss} = 0.0025$.

A.1   Power factor one at the substation input

The problem is to find the required cosine of phi at the inverter output if the desired cosine of phi at delivery point is one, with the delivery point defined as the substation input.

$$\textbf{Goal: } \cos(\varphi_{\text{d, desired}}) = 1$$

Using the algorithm described in Section 5, it is found that for this example, the cosine of phi at the inverter output should be 0.9973.

To validate this result, we will calculate the cosine of phi at the delivery point, which is equivalent to manually performing the last iteration (albeit with a lower precision). The first step is to compute the plant sizing variables.

1. Using equation (1), compute the rated power of the plant:

$$S_{\text{plant}} = \frac{W_{dc}}{r_{dcac} \cdot \cos \varphi_{inv}} = \frac{500 \times 10^3}{1.2 \cdot 0.9973} = 417.8 \times 10^3\,\text{kVA}$$

2. Compute the number of power stations, using equation (5). Note that the number is floating point instead of an integer, this is not an issue since this result is used to compute total losses at each point in the chain.

$$n_{ps} = \frac{S_{\text{plant}}}{S_{inv} \cdot n_{\text{inv per ps}}} = \frac{417.8 \times 10^3}{1500 \cdot 2} = 139.26$$

3. Compute the losses in the medium voltage system cables, using equation (13), at power station rated power.

$$\rho = \rho_{20} \cdot (1 + \alpha(\theta_i - 20)) = \frac{1}{35} \cdot 10^{-6} \cdot (1 + 0.00392 \cdot (90 - 20)) = 0.0364 \Omega mm^2/m$$

$$R_{\text{mv}} = \frac{\rho_{20}}{\Phi} = \frac{0.0364}{400 \cdot 10^{-6}} = 9.1 \cdot 10^{-5} \Omega/m$$

$$L = \left(K + 0.2 \cdot \ln \frac{2 \cdot S}{d} \right) \cdot 10^{-6} = \left(0.05 + 0.2 \cdot \ln \frac{2 \cdot 2 \cdot d}{d} \right) \cdot 10^{-6} = 0.3273 mH/m$$

$$X = 2\pi f L = 2\pi \cdot 50 \cdot 0.3273 \cdot 10^{-6} = 10.28 \cdot 10^{-5} \Omega/m$$

$$W_c = \sqrt{\frac{(S_{\text{rated}} \cdot u_{\text{loss}})^2}{1 + \left(\dfrac{R \cos \varphi_{\text{inv}}}{X \sin \varphi_{\text{inv}}} \right)^2} \cdot \frac{R \cos \varphi_{\text{inv}}}{X \sin \varphi_{\text{inv}}}}$$

$$= \sqrt{\frac{(1500 \cdot 2 \cdot 0.005)^2}{1 + \left(\dfrac{9.1 \cdot 10^{-5} \cdot 0.9973}{10.28 \cdot 10^{-5} \cdot 0.0734} \right)^2} \cdot \frac{9.1 \cdot 10^{-5} \cdot 0.9973}{10.28 \cdot 10^{-5} \cdot 0.0734}} = 14.95\,\text{kW}$$

$$Q_c = \sqrt{\frac{(S_{\text{rated}} \cdot u_{\text{loss}})^2}{1 + \left(\dfrac{R \cos \varphi_{\text{inv}}}{X \sin \varphi_{\text{inv}}} \right)^2}} = \sqrt{\frac{(1500 \cdot 2 \cdot 0.005)^2}{1 + \left(\dfrac{9.1 \cdot 10^{-5} \cdot 0.9973}{10.28 \cdot 10^{-5} \cdot 0.0734} \right)^2}} = 1.24\,\text{kVAr}$$

4. Compute the active power losses in the power station transformer, using equations (6) and (7), with rated power of the transfomer $S_{\text{rated, ps}} = S_{\text{inv}} \cdot n_{\text{inv per ps}} = 1500 \cdot 2 = 3000\,\text{kVA}$.

$$W_{\text{iron}} = F_{\text{iron}} \cdot S_{\text{rated}} = 0.001 \cdot 3000 = 3\,\text{kW}$$

$$W_{\text{copper}} = F_{\text{copper}} \cdot S_{\text{rated}} \cdot x^2 = 0.005 \cdot 3000 \cdot 0.88^2 = 11.6\,\text{kW}$$

5. Compute the no load current of the transformer, using equation (9). Remember that the equation is formulated using rated power in MVA.

$$i_0 = 0.0421 \cdot \left(\log(S_{\text{rated}} \cdot 10^{-6}) \right)^2 - 0.4384 \cdot \log(S_{\text{rated}} \cdot 10^{-6}) + 1.6064 =$$

$$0.0421 \cdot \left(\log(3000 \cdot 10^3 \cdot 10^{-6}) \right)^2 - 0.4384 \cdot \log(3000 \cdot 10^3 \cdot 10^{-6}) + 1.6064 = 1.609$$

6. Compute the reactive power losses in the transformer, using equations (8) and (10). The short circuit voltage percentage is 8 %.

$$Q_{\text{iron}} = \sqrt{\left(\frac{i_0}{100} \cdot S_{\text{rated}} \right)^2 - W_{\text{iron}}^2} = \sqrt{\left(\frac{1.609}{100} \cdot 3000 \right)^2 - 3^2} = 48.17\,\text{kVAr}$$

$$Q_{\text{copper}} = x^2 \cdot \sqrt{\left(\frac{u_{\text{sc}}}{100} \cdot S_{\text{rated}} \right)^2 - W_{\text{copper}}^2} = 0.88^2 \cdot \sqrt{\left(\frac{8}{100} \cdot 3000 \right)^2 - 11.6^2} = 185.64\,\text{kVAr}$$

7. Compute the combined active and reactive power losses, using equations (20) and (21), respectively. The loses in the inverter cable, high voltage line, and in the substation transformer, as well as the reactive loss in the medium voltage cable, have been removed from

the equation for clarity.

$$W_{\text{total}} = W_{\text{c, mv}} \cdot n_{\text{ps}} + W_{\text{t, ps}} \cdot n_{\text{ps}} =$$
$$14.95 \cdot 139.26 + (3 + 11.6) \cdot 139.26 = 4115.1 \, \text{kW}$$
$$Q_{\text{total}} = Q_{\text{t, ps}} \cdot n_{\text{ps}} = (48.17 + 185.64) \cdot 139.26 = 32\,560.4 \, \text{kVAr}$$

8. Finally, using equation (22), compute the phi angle at the delivery point, and the cosine of that angle. The sine of the angle resulting from the inverter cosine of phi is 0.0734.

$$\varphi_d = \arctan\left(\frac{S_{\text{plant}} \cdot \sin\varphi_{\text{inv}} - Q_{\text{total}}}{S_{\text{plant}} \cdot \cos\varphi_{\text{inv}} - W_{\text{total}}}\right) =$$
$$\arctan\left(\frac{417.8 \times 10^3 \cdot 0.0734 - 32560.4}{417.8 \times 10^3 \cdot 0.9973 - 4115.1}\right) = -0.263 \, \text{deg}$$
$$\cos(\varphi_d) = \cos(-0.263) = 0.9999$$

A.2   Power factor 0.95 at the substation input

For this example, the desired cosine of phi is 0.95 at the substation input.

$$\textbf{Goal:} \cos(\varphi_{\text{d, desired}}) = 0.95$$

Using the algorithm described in Section 5, it is found that the cosine of phi at the inverter output should be 0.927. Repeating the steps outlined in Subsection A.1:

1. Using equation (1), compute the rated power of the plant. As expected, the power plant will have a higher apparent rated power, which will result in more inverters.

$$S_{\text{plant}} = \frac{W_{\text{dc}}}{r_{\text{dcac}} \cdot \cos\varphi_{\text{inv}}} = \frac{500 \times 10^3}{1.2 \cdot 0.9273} = 449.48 \times 10^3 \, \text{kVA}$$

2. Compute the number of power stations, using equation (5).

$$n_{\text{ps}} = \frac{S_{\text{plant}}}{S_{\text{inv}} \cdot n_{\text{inv per ps}}} = \frac{449.48 \times 10^3}{1500 \cdot 2} = 149.83$$

3. Compute the losses in the medium voltage system cables, using equation (13), at power station rated power.

$$\rho = \rho_{20} \cdot (1 + \alpha(\theta_i - 20)) = \frac{1}{35} \cdot 10^{-6} \cdot (1 + 0.00392 \cdot (90 - 20)) = 0.0364 \Omega mm^2/m$$

$$R_{mv} = \frac{\rho_{20}}{\Phi} = \frac{0.0364}{400 \cdot 10^{-6}} = 9.1 \cdot 10^{-5} \Omega/m$$

$$L = \left(K + 0.2 \cdot \ln \frac{2 \cdot S}{d}\right) \cdot 10^{-6} = \left(0.05 + 0.2 \cdot \ln \frac{2 \cdot 2 \cdot d}{d}\right) \cdot 10^{-6} = 0.3273 mH/m$$

$$X = 2\pi f L = 2\pi \cdot 50 \cdot 0.3273 \cdot 10^{-6} = 10.28 \cdot 10^{-5} \Omega/m$$

$$W_c = \sqrt{\frac{(S_{rated} \cdot u_{loss})^2}{1 + \left(\frac{R\cos\varphi_{inv}}{X\sin\varphi_{inv}}\right)^2}} \cdot \frac{R\cos\varphi_{inv}}{X\sin\varphi_{inv}}$$

$$= \sqrt{\frac{(1500 \cdot 2 \cdot 0.005)^2}{1 + \left(\frac{9.1 \cdot 10^{-5} \cdot 0.927}{10.28 \cdot 10^{-5} \cdot 0.375}\right)^2}} \cdot \frac{9.1 \cdot 10^{-5} \cdot 0.927}{10.28 \cdot 10^{-5} \cdot 0.375} = 13.64\,\text{kW}$$

$$Q_c = \sqrt{\frac{(S_{rated} \cdot u_{loss})^2}{1 + \left(\frac{R\cos\varphi_{inv}}{X\sin\varphi_{inv}}\right)^2}} = \sqrt{\frac{(1500 \cdot 2 \cdot 0.005)^2}{1 + \left(\frac{9.1 \cdot 10^{-5} \cdot 0.927}{10.28 \cdot 10^{-5} \cdot 0.375}\right)^2}} = 6.23\,\text{kVAr}$$

4. The losses in the power station transformer remain the same.

5. Compute the combined active and reactive power losses, using equations (20) and (21), respectively.

$$W_{total} = W_{c,\,mv} \cdot n_{ps} + W_{t,\,ps} \cdot n_{ps} =$$
$$13.64 \cdot 149.83 + (3 + 11.6) \cdot 149.83 = 4231.2\,\text{kW}$$
$$Q_{total} = Q_{t,\,ps} \cdot n_{ps} = (48.17 + 185.64) \cdot 149.83 = 35\,031.75\,\text{kVAr}$$

6. Finally, using equation (22), compute the phi angle at the delivery point, and the cosine of that angle. The sine of the angle resulting from the inverter cosine of phi is 0.375.

$$\varphi_d = \arctan\left(\frac{S_{plant} \cdot \sin\varphi_{inv} - Q_{total}}{S_{plant} \cdot \cos\varphi_{inv} - W_{total}}\right) =$$
$$\arctan\left(\frac{449.48 \times 10^3 \cdot 0.375 - 35031.75}{449.48 \times 10^3 \cdot 0.927 - 4231.2}\right) = 17.95\,\text{deg}$$
$$\cos(\varphi_d) = \cos(17.95) = 0.951$$

The error in the final value of the cosine of phi can be expected due to rounding.